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Abstract

The aim of this paper is to generate a new copula based Value at Risk (VaR) approach that can be applied to high-dimensional real world portfolios. Current VaR copula models typically only can deal with portfolios consisting of just a few risk factors. They are, therefore, not suitable for practical applications. This paper tries to fill this gap by presenting a new parsimonious and fast calibration algorithm for the Student $t$ copula model. The new approach provides for the first time the possibility to generate VaR estimates based on Student $t$ copulas for high-dimensional real world portfolios.

A portfolio of 21 different financial assets and three additional VaR models (Variance-Covariance, Gaussian copula, and historical simulation) are used to evaluate the suitability of this new Student $t$ copula approach. Almost 20 years of data are used to conduct an out-of-sample hit test based on a rolling window of 250 trading days for model calibration. The results of the hit test reveal that the model performance is highly affected by volatility clustering. Thus, all models perform poorly based on empirical returns, a fact that can be attributed to the underestimation of risk during the financial crisis in 2008. The new Student $t$ copula approach and the historical simulation model perform best, whereas the Variance-Covariance model performs worst in this environment.

Accounting for volatility-clustering and applying the models on GARCH(1,1)-innovations rather than on empirical returns considerably improves the performance. Overall, the weaknesses of the Variance-Covariance model stems from three sources: (a) An inappropriate modeling of (univariate) return distributions, (b) an inappropriate modeling of the ‘dependence structure’ (i.e. the copula), and (c) not accounting for volatility clustering. The proposed new Student $t$ copula approach tends to overcome these weaknesses when volatility clustering is accounted for. It is, therefore, a quite promising parametric model alternative for the Variance-Covariance model.
1 Introduction

This article presents a new parsimonious approach to estimate a Student $t$ copula for high dimensional data sets in the context of a one-day market-risk management. Current Value at Risk (VaR) copula models typically only can deal with portfolios consisting of just a few risk factors. Thus, they are not suitable for practical applications. Our new approach provides for the first time the possibility to generate VaR estimates based on Student $t$ copulas for high-dimensional real world portfolios.

Nowadays the most widely used market risk models assume a multivariate Gaussian distribution of asset returns (‘Variance-Covariance model’). Various studies have shown, however, that these models underestimate risk measures like the VaR. This is due to two facts: First, returns of financial assets typically do not have a Gaussian distribution but display ‘heavy tails’, implying higher risk. Second, the ‘dependence structure’ – the so-called copula – does not correspond to a Gaussian distribution. Typically, the probability of joint extreme losses observable empirically is higher than that implied by a Gaussian copula.

The Student $t$ copula seems to be an appropriate alternative as it assigns a higher probability to these joint extreme losses of financial assets. Also, it will be well understood by practitioners as it has a correlation matrix as a first parameter (like the Gaussian copula) and only one additional parameter – the degrees of freedom ($\nu$) – that controls the probability of joint extreme losses. This intuitive comprehensibility is in contrast to other widely applied copulas like the BB1, Clayton or Gumbel copulas or to more advanced copula-models like vines. Furthermore, for the latter models the calibration is comparatively complicated and very time-consuming.

In addition, the calibration of a Student $t$ copula is computationally very intense and, hence, time-consuming when the ‘standard’ calibration procedure is used. This leads us to propose a parsimonious copula-parameter calibration process where the parameter $\nu$ is estimated on the basis of bivariate data-pairs.

We conduct an out-of-sample hit test to evaluate the accuracy of the estimation of a one-day 99% Value-at-Risk of a 21-dimensional equally weighted portfolio, using a data history of almost 20 years.
The following models are employed, using a rolling window of 250 trading days: (i) the ‘standard’ Variance-Covariance model, (ii) a Gaussian copula model where the marginal distributions are modeled as empirical distributions based on the 250 most recent observations, (iii) a Student $t$ copula model with degree of freedom parameter $\nu$ calibrated based on bivariate data, and (iv) a classical historical simulation approach.

We find that the benchmark Variance-Covariance model severely underestimates the Value at Risk. The model based on the Gaussian copula leads to an improvement. This is due to a more realistic modelling of the (univariate) asset returns. The copula for the asset returns is the same as in the Variance-Covariance model. Using a Student $t$ copula leads to further improvements. The results of the simple non-parametric historical simulation model are similar to those of the Student $t$ copula model.

However, all models fail to predict the Value at Risk accurately during the financial crisis in 2008. We additionally test our model on the innovations of a GARCH(1,1)-process applied on the original data set. Here, the models perform much better and a Kupiec test rejects the null hypothesis of a correct specification only for the Variance-Covariance model. Hence, it seems very important to account for volatility clustering which can be identified as a third source for underestimation of risk.

The remainder of this article is structured as follows: Section 2 presents the models used for the hit test and section 3 depicts the data base. The results of the hit test are discussed in section 4 and section 5 concludes.

2 Models employed

Several models are used to estimate the one-day 99%-Value-at-Risk of an equally weighted portfolio consisting of 21 long positions in financial assets (see section 3). Specifically, for each trading day we estimate the 1%-quantile of portfolio-returns always using a rolling window of 250 trading days for calibration and conduct a hit test later on.
2.1 Variance-Covariance model

The ‘Variance-Covariance model’ assumes a multivariate Gaussian distribution of risk factor changes (i.e. asset returns). This model is still the most widely used model to assess market risk and is implemented in software solutions like, e.g. RiskMetrics (see Mina and Xiao [34]). The Variance-Covariance model typically ignores (for a 1-day horizon) return expectations and only considers the variance-covariance matrix of portfolio asset returns. The 1%-quantile of the portfolio returns is hence estimated as

\[
\Phi^{-1}(0.01) \sqrt{w^\top \Sigma w},
\]

where \( \Phi^{-1} \) is the quantile function of the univariate standard normal distribution function, \( w \) is a column vector containing the portfolio weights\(^1\) and \( \Sigma \) represents the sample variance-covariance matrix of risk factor changes based on the 250 most recent observations.

We use a rolling window of 250 trading days to estimate the 1%-quantile of the daily portfolio return distribution. Then we conduct an out-of-sample hit test in which we compare the (realized) next day’s \((t = 251)\) portfolio return with our 1%-quantile estimate. If the return is below the 1%-quantile – i.e. the portfolio value drop exceeds the 99%-Value-at-Risk – we observe a ‘hit’.

2.2 Copula-based methods

In the context of modelling the joint distribution of portfolio returns consisting of several financial assets, a copula approach allows to decompose this task into two separate steps: First, modelling of the univariate return distributions of financial assets and, second, modelling of their copula (i.e. their ‘dependence structure’). The term ‘copula’ was introduced by Sklar [39] in 1959 (a similar concept for modelling dependence structures of joint distributions was independently proposed by Höffding [19] some twenty years earlier).

\(^1\)In our case \( w = \begin{pmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{pmatrix} \).
Copulas are functions that combine or couple (univariate) ‘marginal distributions’ to a multivariate joint distribution. Sklar’s theorem states that a $n$-dimensional joint distribution function $F(x)$, where $x = (x_1, x_2, \ldots, x_n)$, may be expressed in terms of the joint distribution’s copula $C$ and its marginal distribution functions $F_1, F_2, \ldots, F_n$ as

$$ F(x) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)), \quad x \in \mathbb{R}^n. \quad (2) $$

The copula function $C$ is by itself a multivariate distribution with uniform marginal distributions on the interval $U_1 = [0, 1], C : U_1^n \to U_1$. In this paper we restrict ourselves to two elliptical copulas, the Gaussian copula and its generalisation, the Student $t$ copula. More information on these two copulas is given in the subsequent two subsections.

One of the main advantages of copula-based approaches is that they allow for an arbitrary modelling of the marginal distributions – in our case the return distribution of financial assets. This is in contrast to the widely used Variance-Covariance model that assumes Gaussian marginal distributions. Dating back to the 1960s, empirical studies (see, e.g., Fama [14], or Mandelbrot [31]) have found that the assumption of Gaussian distributions of financial asset returns can be rejected, as financial asset returns typically display ‘heavy tails’, i.e. they are leptokurtic. If asset returns are leptokurtic and one assumes a Gaussian distribution one typically underestimates risk.

Another aspect of copula based models is that they allow for an explicit modelling of the ‘dependence structure’ (i.e. the copula) between returns of financial assets. Recent studies have shown that the Gaussian copula (which is implicitly assumed in the Variance-Covariance model) underestimates the probability of joint extreme co-movements of risk factor changes. In the context of market risk modelling this means that the probability of joint severe losses for financial assets is underestimated. The Student $t$ copula seems a suitable candidate in a first step to improve market risk models in this context. It allows for modelling the probability mass that is assigned to

\footnote{Later studies confirming these findings are, e.g., Bekaert and Harvey [5], Hassan et al. [18], Husain [21], Laurence [29], Liang et al. [30], Miljković and Radović [33], and Sarma [36].}

\footnote{See, e.g., Aussenegg and Cech [3], Cech [8], Cech and Fortin [9], Cech and Fortin [10], Fortin and Kuzmics [15], Hurd et al. [20], Jondeau and Rockinger [23], Junker et al. [24], Kang [25], Kat and Palaro [26], Kole et al. [27], Mashal and Zeevi [32], and Patton [35].}
joint extreme co-movements. Furthermore, the Student $t$ copula will be well understood by practitioners that have already worked with models assuming multivariate normality, as the familiar notion of the correlation matrix is pertained and as only one parameter is additionally introduced. This is in contrast to other widely used copulas in finance like, e.g., the BB1 copula and its two special cases, the Clayton and Gumbel copula. Their copula parameters cannot be interpreted in a way similar to those of the Student $t$ or Gaussian copula. They are also less flexible in their ‘standard’ version concerning the modeling of higher dimensional dependence structures (more than two dimensions). Here, more advanced approaches like nested copula constructions or pair-copula constructions (also referred to as vines) seem advisable (for a review of these models see, e.g., Berg and Aas [6]). However, these pair-constructions are difficult to implement and take a very long time to calibrate in a high-dimensional problem setting.\footnote{To optimise the calibration one ideally would have to estimate the parameters on all permutations of the dimensions of the data set and then select the model with the best fit.}

Our copula parameters are estimated from the empirical return observations using the pseudo-log-likelihood method (see Genest and Rivest [17]). In this approach, no assumptions on the specific functional form of the marginal distributions have to be made. Before conducting a maximum likelihood estimation, the empirical joint observations $\hat{x}_t = (\hat{x}_{1,t}, \hat{x}_{2,t}, \ldots, \hat{x}_{n,t})$, $t \in \{1, 2, \ldots, T\}$, are transformed into so-called ‘pseudo-observations’ $\hat{u}_t = (\hat{u}_{1,t}, \hat{u}_{2,t}, \ldots, \hat{u}_{n,t})$:

$$\hat{u}_{i,t} = \frac{1}{T} \sum_{s=1}^{T} 1_{\hat{x}_{i,s} \leq \hat{x}_{i,t}},$$

where $1_{\hat{x}_{i,s} \leq \hat{x}_{i,t}}$ is an indicator function that takes a value of 1 if $\hat{x}_{i,s} \leq \hat{x}_{i,t}$ and a value of 0 otherwise. In other words, eq. 3 is computed by dividing the ranks of the observations by $T$. In our estimation procedure we slightly adjust the computation of the pseudo-observations by using tied-ranks rather than ranks and by dividing by $(T + 1)$ rather than by $T$.\footnote{Dividing by $(T + 1)$ rather than by $T$ is an approach that has been proposed by Demarta and McNeil [12]. This approach keeps the pseudo-observations away from the boundaries of the unit cube where the density of many copulas take infinite values.} Employing these pseudo-observations, the copula parameters are estimated via maximum likelihood estimation.
The pseudo-log-likelihood approach is nowadays the most commonly used method as it achieves a better fit than methods that use correlation measures such as ‘Kendall’s tau’ or ‘Spearman’s rho’ to estimate copula parameters (see, e.g., Genest et al. [16]). Another widely used method is the so-called IFM (Inference Function for Margins) method. The drawback of this approach is that the functional forms of the marginal distributions have to be assumed. Scaillet and Fermanian [37] conduct a simulation study to assess the impact of misspecified marginal distributions and find that the errors for the copula parameter estimates can be very large if the marginal distributions are misspecified.

2.2.1 Meta-Gaussian model

The Gaussian copula is the copula that is implied by a multivariate Gaussian distribution (normal distribution). A multivariate Gaussian distribution can be regarded as a set of marginal (univariate) Gaussian distributions that are coupled with a Gaussian copula. Arbitrary marginal distributions that are coupled with a Gaussian copula are referred to as meta-Gaussian distributions. A Gaussian copula has only one parameter $P_G$ (capital ‘Rho’ subscript $G$), the correlation matrix.

Based on the 250 most recent set of pseudo-observation (see eq. 3), we estimate the copula-parameter $P_G$, using a maximum log-likelihood estimation procedure:

$$\hat{P}_G = \arg\max_{P_G} \sum_{t=1}^{250} \left[ \ln \phi_{P_G} \left( \Phi^{-1}(\hat{u}_t) \right) + \sum_{i=1}^{21} \ln \left( \frac{1}{\phi(\Phi^{-1}(u_{i,t}))} \right) \right], \quad (4)$$

where $\phi_{P_G}$ is the probability density function of a multivariate standard normal distribution with correlation matrix $P_G$, $\phi$ is the probability density function of the univariate standard normal distribution, $\hat{u}_t$ is a vector of estimated ’pseudo-observations’, and $\Phi^{-1}$ is the functional inverse of the univariate standard normal cumulative distribution function.

In a second step we use the estimated copula parameter $\hat{P}_G$, obtained from eq. 4, and a rolling window of the 250 most recent return observations of 21 financial assets, to simulate a Gaussian copula and then simulate the

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6For the density of a Gaussian copula see, e.g., Cherubini et al. [11], pp. 147f.
joint return distribution of all 21 financial assets. First, we simulate 10,000 scenarios of a Gaussian copula. Second, we compute – according to the scenarios of the Gaussian copula simulation – scenarios of a 21-dimensional asset return distribution, where the marginal distributions are modeled as Gaussian kernel smoothed distributions\(^7\). Third, the 1%-quantile of the portfolio returns is estimated from these 10,000-scenario portfolio returns. Again, a hit test is conducted.

2.2.2 Parsimonious estimation of a multidimensional meta-Student \(t\) model

Student \(t\) copulas are a generalization of Gaussian copulas. They are the copulas implied by a multivariate Student \(t\) distribution which can be considered as a set of marginal (univariate) Student \(t\) distributions that are coupled with a Student \(t\) copula. Arbitrary marginal distributions that are coupled with a Student \(t\) copula are referred to as meta-Student \(t\) distributions. Student \(t\) copulas have two parameters: the correlation matrix \(P_t\) and a scalar parameter \(\nu\), the degrees of freedom, which controls the probability mass assigned to joint extreme co-movements of risk factor changes (i.e. asset returns). Compared to a Gaussian copula, the Student \(t\) copula assigns a higher probability to joint extreme co-movements. The lower the parameter \(\nu\), the higher this probability is. Gaussian copulas can be regarded as a special case of Student \(t\) copulas, with \(\nu \to \infty\).\(^8\)

In a standard estimation process the Student \(t\) copula parameters \(P_t\) and \(\nu\) are calibrated using a maximum log-likelihood estimation procedure\(^9\), here based on the 250 most recent set of pseudo-observation (see eq. 3):

\[
(\hat{\nu}, \hat{P}_t) = \underset{\nu, P_t}{\text{argmax}} \sum_{t=1}^{250} \left[ \ln f_{\nu, P_t} \left( t_{\nu}^{-1}(\hat{u}_t) \right) + \sum_{i=1}^{21} \ln \left( \frac{1}{f_{\nu} \left( t_{\nu}^{-1}(u_{i,t}) \right)} \right) \right],
\]

where \(\ln f_{\nu, P_t}\) is the probability density function of a multivariate Student \(t\) distribution with \(\nu\) degrees of freedom and correlation matrix \(P_t\), \(f_{\nu}\) is the probability density function of a univariate Student \(t\) distribution with \(\nu\) degrees of freedom, and \(t_{\nu}^{-1}\) is the functional inverse of the univariate Student

\(^7\)See, e.g., Silverman [38] pp 34ff.

\(^8\)For more information on Student \(t\) copulas see, e.g., Demarta and McNeil [12].

\(^9\)For the density of a Student \(t\) copula see, e.g., Cherubini et al. [11], p. 148.
Table 1: Computing time

This table presents the computing time in seconds for the calibration of a Gaussian and a Student $t$ copula on 250 sets of randomly generated $d$-dimensional data.

<table>
<thead>
<tr>
<th>$d$</th>
<th>Gaussian</th>
<th>Student $t$</th>
<th>$d$</th>
<th>Gaussian</th>
<th>Student $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.11</td>
<td>1.48</td>
<td>12</td>
<td>0.00</td>
<td>3162.40</td>
</tr>
<tr>
<td>3</td>
<td>0.00</td>
<td>0.94</td>
<td>13</td>
<td>0.00</td>
<td>648.88</td>
</tr>
<tr>
<td>4</td>
<td>0.02</td>
<td>1.81</td>
<td>14</td>
<td>0.02</td>
<td>6715.80</td>
</tr>
<tr>
<td>5</td>
<td>0.02</td>
<td>6.01</td>
<td>15</td>
<td>0.02</td>
<td>9949.88</td>
</tr>
<tr>
<td>6</td>
<td>0.00</td>
<td>11.93</td>
<td>16</td>
<td>0.00</td>
<td>13421.60</td>
</tr>
<tr>
<td>7</td>
<td>0.00</td>
<td>227.84</td>
<td>17</td>
<td>0.00</td>
<td>17196.83</td>
</tr>
<tr>
<td>8</td>
<td>0.00</td>
<td>443.76</td>
<td>18</td>
<td>0.00</td>
<td>3755.87</td>
</tr>
<tr>
<td>9</td>
<td>0.00</td>
<td>99.42</td>
<td>19</td>
<td>0.00</td>
<td>26490.97</td>
</tr>
<tr>
<td>10</td>
<td>0.00</td>
<td>157.70</td>
<td>20</td>
<td>0.00</td>
<td>43669.88</td>
</tr>
<tr>
<td>11</td>
<td>0.00</td>
<td>1946.48</td>
<td>21</td>
<td>0.02</td>
<td>69574.82</td>
</tr>
</tbody>
</table>

$t$ cumulative distribution function with $\nu$ degrees of freedom.

While a Student $t$ copula allows for a more realistic modelling of extreme joint co-movements, the calibration of a Student $t$ copula takes much longer than the calibration of a Gaussian copula. This is specifically true for high-dimensional copulas (i.e. a portfolio of many assets) and if $\nu$ is large. Table 1 shows the computing time for (only) one scenario (using a ‘standard’ personal computer) for the calibration of a Gaussian and a Student $t$ copula on randomly generated data of 250 sets with different dimensions. The generated data are Gaussian copula random variables, which implies that $\nu \to \infty$ if a Student $t$ copula is calibrated on this data. As can be seen, the Student $t$ copula calibration takes too long for high dimensions to be implemented in practice, e.g. more than 19 hours for a 21-dimensional Student $t$ copula.

This fact motivates our parsimonious approach where the Student $t$ copula parameter $\nu$ is estimated based on bivariate pairs of observations. Specifically, we employ the following algorithm:

1. Construct bivariate pairs of the time series. Having a $d$-dimensional time series, this will result in $\frac{d(d-1)}{2}$ bivariate pairs.\(^{10}\) For each of these

\(^{10}\)In our case, where $d = 21$, this results in 210 pairs.
pairs calibrate a Student $t$ copula and store the copula parameter $\hat{\nu}_i$.

2. Use the median of the parameters $\hat{\nu}_i$ as the parameter $\hat{\nu}$ for the $d$-dimensional Student $t$ copula.

3. Approximate the correlation matrix ($P$) by calibrating a Gaussian copula on the $d$-dimensional data and use the resulting Gaussian copula parameter $\hat{\nu}_G$ as a proxy for the Student $t$ copula parameter $\hat{\nu}$.\textsuperscript{11}

This procedure helps to speed up the calibration process considerably. In our 21-dimensional case, the calibration takes less than 1 minute for a window of 250 trading days on a ‘standard’ personal computer. This is more than 10,000 times faster compared to a traditional 21-dimensional Student $t$ copula estimation (see Table 1).

Again, a simulation with 10,000 scenarios is conducted where a Student $t$ copula and the assets’ marginal returns (again modeled as empirical Gaussian kernel smoothed distributions) are generated. Then, the 1%-quantile of portfolio returns is estimated from the 10,000 scenario portfolio returns.

We also consider a variant of the above parsimonious estimation of $\nu$, where we only use a rolling window of 50 trading days (rather than 250 trading days) for the estimation of the parameter $\nu$. This allows $\nu$ to adapt more quickly as we lower the impact of the so-called ‘ghost effect’.\textsuperscript{12} As in the case of the meta-Gaussian model, a hit test is conducted to investigate the quantile estimates for both meta-Student $t$ model versions (250 and 50 trading days to estimate parameter $\nu$).

### 2.3 Historical simulation

The historical simulation approach\textsuperscript{13} is a non-parametric procedure to estimate the Value at Risk of a portfolio, i.e. a particular quantile of the portfolio’s return distribution. Again, we use a rolling window of 250 trading days to estimate the 1% portfolio return quantile. To do this, we apply the portfolio weights (in our case of an equally weighted portfolio of 21 assets,

\textsuperscript{11}This is a conservative approach. Empirical evidence suggests that if $\hat{\nu}$ is low, the elements of $\hat{\nu}_G$ are smaller (in absolute terms) compared to the elements of $\hat{\nu}_t$ calibrated on the same data. As $\nu \to \infty$, $\hat{\nu}_t$ and $\hat{\nu}_G$ are identical.

\textsuperscript{12}Compare e.g. Alexander [1] pp. 52-53 on the ghost effect for volatility estimates.

$w_i = \frac{1}{21} \forall i$ to the historic asset returns of the 250 most recent trading days and compute the portfolio return that would have resulted for each of these 250 days for the specified portfolio weights. From these 250 ‘historically simulated’ portfolio returns we compute the 1%-quantile for which a hit test is conducted. Hence, our estimate is based on the second- and third-lowest return of our small sample of only 250 scenarios (as the second lowest return represents the 0.8%-quantile and the third lowest return represents the 1.2%-quantile).

3 Data

The data base consists of daily observations (log-returns based on closing prices) of 21 financial assets from August 1st, 1990 to July 30th, 2010 (4,997 daily returns). These financial assets can broadly be classified into four asset-classes: (a) foreign exchange (3 assets), (b) blue-chip stocks (6 assets) and stock-indices (3 assets), (c) commodities (3 assets), and (d) fixed-income instruments with different maturities (6 assets). In particular, the 21 assets are: USD per EUR, USD per GBP, USD per JPY, stocks of Boeing, Walt Disney, IBM, Verizon Communications, Wal-Mart Stores, Exxon Mobile, the S&P 500 index, the DAX 30 index, the Nikkei 225 index, West Texas Intermediate (WTI) crude oil, gold, palladium, as well as 3-month, 1-, 2-, 3-, 5-, and 10-year discount factors of the US Treasury yield curve.

Thomson Reuters 3000 Xtra is used as data source for the foreign exchange, stock, stock index, and commodity assets. Discount factors of the US-Treasury yield curve are obtained from the Office of Debt Management (US-Treasury Department).14 Depart from DAX 30 and Nikkei 225 indices, all assets are denominated in USD.15 Returns of the six blue-chip stocks include dividend payments. The Treasury’s yield curve is generated using a quasi-cubic hermite spline function.16 The corresponding inputs to generate

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15To ensure that ‘dependence structure’ estimates are not too much affected by different closing times, most of our assets are based on closing prices between 9:30pm and 10:30pm (CET). Exceptions are WTI crude oil (8:30pm CET) and the two non-US stock indices (DAX at 5:30pm and Nikkei 225 at 8:00am CET)

the yield curve are prices of the most recently auctioned 4-, 13-, 26-, and 52-week US-Treasury bills, the most recently auctioned 2-, 3-, 5-, 7-, and 10-year US-Treasury notes, as well as the most recently auctioned US-Treasury 30-year bond.\footnote{Additional bid yields are used if there is no on-the-run security available for a given maturity (see Treasury Yield Curve Methodology at http://www.treas.gov/offices/domestic-finance/debt-management/interest-rate/yield.shtml.}

Table 2 exhibits main descriptive statistics on daily return distributions for our 21 assets. Mean (median) daily returns are small but positive, except for the Japanese stock market. The return volatility is lowest for fixed-income instruments, and highest for individual stocks and commodities.\footnote{The fixed-income instrument returns are daily relative price changes of corresponding discount factors. They, thus, represent the performance of zero bonds with constant maturities.}

The returns of all assets are symmetric (skewness value near zero) but exhibit partly severe heavy tails (large excess kurtosis). The latter are especially pronounced for money market instruments (US-Treasury short term discount factors) and commodities, but can also be observed for most individual stocks and stock indices.\footnote{This heavy tail characteristics are an indication that risk modeling based on normal distribution assumptions might lead to inefficient results. Using a Jarque-Bera Test (Jarque and Bera [22]), the null hypothesis of normally distributed asset returns can be rejected for all 21 assets at the 1% significance level.}

Figure 1 presents in this context a graphical comparison of the 21 return distributions. It also reveals the diversity in dispersion of the asset classes used.

4 Results

We conduct a hit test to assess the appropriateness of our models. The Variance-Covariance model is considered as benchmark. We make 4,746 out-of sample VaR forecasts\footnote{Although we have 4,997 sets of joint return observations we can only conduct and evaluate 4,746 out-of-sample forecasts as we need the first 250 observations for calibration of the first forecast and as we need the last observation to evaluate the last forecast (4,746 = 4,997 − 250 − 1).} and count how many times the portfolio return is below the 1%-quantile forecast. If a model is correctly specified one would expect to observe roughly 47 hits.
### Table 2: Descriptive Statistics

Main descriptive statistics of the daily return distribution for 21 assets. The distribution parameters are generated based on the total observation period used (August 1, 1990 to July 30th, 2010, 4997 daily returns).

<table>
<thead>
<tr>
<th>Asset</th>
<th>mean</th>
<th>median</th>
<th>10%-quant.</th>
<th>90%-quant.</th>
<th>min</th>
<th>max</th>
<th>std. dev.</th>
<th>skew</th>
<th>e. kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>USD per EUR</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>USD per GBP</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>USD per JPY</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Boeing</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Walt Disney</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
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</tr>
<tr>
<td><strong>IBM</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Verizon Comm.</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Wal-Mart</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Exxon Mobil</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>SP500</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>DAX30</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Nikkei225</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Oil (WTI)</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Gold</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Palladium</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>3M US Treas.</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>1Y US Treas.</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>2Y US Treas.</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>3Y US Treas.</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>10Y US Treas.</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>0.00%</td>
<td>0.02</td>
<td>1.94</td>
<td>0.66%</td>
<td>-0.02</td>
<td>0.65</td>
</tr>
</tbody>
</table>
Figure 1: Univariate Return Distributions

Box plot of daily returns for all 21 assets. EUR, GBP, and JPY represent the USD per EUR, the USD per GBP, and the USD per JPY exchange rate, respectively. SP is the S&P 500, DAX the DAX 30, and Nikkei the Nikkei 225 stock index, respectively. Oil is the abbreviation for West Texas Intermediate (WTI) crude oil. The last six assets illustrate US-Treasury fixed income instruments in the form of discount factors (zero bonds) with (constant) maturities of 3 months, 1-, 2-, 3-, 5-, and 10-years. In the box plot, each box has lines at the lower quartile (lower end of the box), median (inside the box), and upper quartile (upper end of the box) of the distributions. From each end of a box, whiskers extend to values that are 1.5 times the interquartile range below respectively above the first and third quartile. Any values beyond the end of the whiskers are outliers that are displayed with a + sign. Note that one extreme return outlier of WTI crude oil is not contained in the graph (-40.7% on January 17th, 1991, the start of the removal of the Iraqi invasion force from Kuwait).
Table 3: Results of the Kupiec hit test

The Kupiec hit test is based on \( n = 4,746 \) daily return observations (August 1\(^{st}\), 1991 until July 20\(^{th}\), 2010) of our equally weighted portfolio consisting of 21 assets.

<table>
<thead>
<tr>
<th>Model</th>
<th># hits</th>
<th>perc. hits</th>
<th>Kupiec p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance-Covariance</td>
<td>91</td>
<td>1.92%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Meta-Gaussian</td>
<td>74</td>
<td>1.56%</td>
<td>0.03%</td>
</tr>
<tr>
<td>Meta-Student ( t [n(\nu) = 250] )</td>
<td>69</td>
<td>1.45%</td>
<td>0.33%</td>
</tr>
<tr>
<td>Meta-Student ( t [n(\nu) = 50] )</td>
<td>66</td>
<td>1.39%</td>
<td>1.07%</td>
</tr>
<tr>
<td>Historical Simulation</td>
<td>66</td>
<td>1.39%</td>
<td>1.07%</td>
</tr>
</tbody>
</table>

Table 3 shows the number of hits per model (i.e. the number of times the loss exceeds the model’s VaR-estimate), the percentage of hits (which should approximate 1% for a correctly specified model) and the p-value of a Kupiec test (Kupiec [28]), testing the null hypothesis of a correctly specified model.

The Variance-Covariance model has 91 hits – almost twice as much as anticipated for a correctly specified model. The Kupiec test suggests that there is almost no doubt that the Variance-Covariance model is not correctly specified. Using our meta-Gaussian model, i.e. modelling the marginal distributions on the basis of the univariate empirical return distributions, but assuming the same ‘dependence structure’ as in the Variance-Covariance model (i.e. a Gaussian copula), leads to a strong reduction of the number of hits (from 91 to 74). Still, the number of hits is more than 50% higher than expected for a correctly specified model and also the Kupiec test (p-value 0.03%) suggests that the model is misspecified. A further comparably modest reduction of the numbers of hits can be achieved by using the Student \( t \) copula instead of the Gaussian copula based on our new parsimonious estimation procedure presented in section 2.2.2. 69 or 66 hits are counted respectively for each of the two variants of our meta-Student \( t \) model. For the latter variant, where only 50 observations are used to calibrate parameter \( \nu \), the Kupiec test p-value is slightly above 1%. The non-parametric historical simulation generates the same number of hits as the second variant of our Student \( t \) copula model (i.e. 66 hits).

Figure 2 reveals the percentage of hits per year for each of our five different models. The underestimation of risk during the peak of the financial
crisis in 2008 can clearly be identified. Especially in this year meta-Student $t$ models perform ‘better’ than meta-Gaussian models, although the underestimation of risk from all models has to be acknowledged. In other years the difference between models is less severe.

For our data sample, the meta-Student $t$ models perform better than the meta-Gaussian model. The main difference between these models is the copula parameter $\nu$, calibrated for the meta-Student $t$ models using the algorithm described in section 2.2.2, while for the meta-Gaussian model $\nu$ equals $\infty$. Mashal and Zeevi [32] point out that the Gaussian copula and the Student $t$ copula are ‘very close’ for $\nu > 100$ and that they are ‘essentially indistinguishable’ for $\nu > 1,000$. From our total of 996,660 estimates of bivariate Student $t$ copulas (i.e. 4,746 trading days times 210 bivariate copula estimates per trading day), 29.0% (26.3%) display estimates of $\nu$ larger than 100 (1,000), when a rolling window of 250 trading days is used. The fraction of estimates of large $\nu$ is even higher when a rolling window of 50 trading days is used. Here, 44.7% (43.8%) display estimates of $\nu$ larger than 100 (1,000). A closer examination of 'small' $\nu$ values reveals that a large fraction of the bivariate estimates of $\nu$ is below 10. A frequency plot of the bivariate estimates of $\nu$ is displayed in figure 3.

Figure 4 displays the median of the distribution of the 210 daily bivariate

![Figure 2: Percentage of hits](image-url)
Figure 3: Distribution of $\nu$

Frequency plots of the estimates of the copula parameter $\nu$ of 996,660 bivariate Student $t$ copula estimates for $\nu < 50$. Note that $\nu$ is smaller than 50 on about 60% of all trading days.
Figure 4: Median of copula parameter $\nu$

Median of the 210 daily estimates of $\nu$ for the bivariate copulas (log-scale), using a rolling window of 250 trading days (top panel) and a rolling window of 50 trading days (bottom panel) to calibrate $\nu$. The depiction is truncated at $\hat{\nu} = 1,000$. 

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Table 4: Results of the Kupiec hit test without the year 2008

The Kupiec hit test is based on $n = 4,495$ daily return observations of our equally weighted portfolio consisting of 21 assets (August 1st, 1991 until July 30th, 2010, without returns in 2008).

<table>
<thead>
<tr>
<th>Model</th>
<th># hits</th>
<th>perc. hits</th>
<th>Kupiec p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance-Covariance</td>
<td>67</td>
<td>1.49%</td>
<td>0.21%</td>
</tr>
<tr>
<td>Meta-Gaussian</td>
<td>54</td>
<td>1.20%</td>
<td>18.85%</td>
</tr>
<tr>
<td>Meta-Student $t \left[ n(\nu) = 250 \right]$</td>
<td>53</td>
<td>1.18%</td>
<td>24.06%</td>
</tr>
<tr>
<td>Meta-Student $t \left[ n(\nu) = 50 \right]$</td>
<td>48</td>
<td>1.07%</td>
<td>65.11%</td>
</tr>
<tr>
<td>Historical Simulation</td>
<td>50</td>
<td>1.11%</td>
<td>45.71%</td>
</tr>
</tbody>
</table>

estimates of $\nu$. These (median) estimates of $\nu$ are used as parameter in our Student $t$ copula model and vary considerably over time. Upon closer inspection of the top panel of figure 4, where the median of the $\nu$-distribution ($\hat{\nu}$) of variant 1 of our model ($n(\nu) = 250$) is shown, we can identify that there are several time periods where a low probability is assigned to joint extreme co-movements of risk factor changes ($\hat{\nu} > 1,000$). These time periods are the second half of 1993, the year 1995 and the last quarter of 1999. The bottom panel shows $\hat{\nu}$ for variant 2 of our model ($n(\nu) = 50$). Here, the variation over time is much more pronounced than in variant 1, as this model can adopt more quickly to changes in the market environment. The top panel of figure 4 further reveals low $\nu$ values (strong co-movements) in volatile periods (turn of the years 1999/2000, 2007/2008, and especially in 2009).

The somewhat unsatisfactory results for the newly proposed Student $t$ copula model approach are mainly due to the underestimation of risk in 2008. Table 4 shows the results for the Kupiec test when the 2008 data of the hit test are excluded. While the null hypothesis of a correct model specification can again be rejected for the Variance-Covariance model at the 1% significance level, it cannot be rejected for the other models. So we confirm that the ability to model marginal distributions in copula-approaches has a large potential to improve Value at Risk models. Again, the Student $t$ copula model performs somewhat better than the Gaussian copula model and, again, the simple historical simulation yields results similar to the copula approaches.
Table 5: Results of the Kupiec hit test applied to GARCH(1,1)-innovations

The Kupiec hit test is based on $n = 4,746$ daily GARCH(1,1)-innovations observations of our equally weighted portfolio consisting of 21 assets (August 1st, 1991 until July 30th, 2010).

<table>
<thead>
<tr>
<th>Model</th>
<th># hits</th>
<th>perc. hits</th>
<th>Kupiec p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance-Covariance</td>
<td>65</td>
<td>1.37%</td>
<td>1.54%</td>
</tr>
<tr>
<td>Meta-Gaussian</td>
<td>52</td>
<td>1.10%</td>
<td>51.42%</td>
</tr>
<tr>
<td>Meta-Student $t \ [n(\nu) = 250]$</td>
<td>43</td>
<td>0.91%</td>
<td>43.71%</td>
</tr>
<tr>
<td>Meta-Student $t \ [n(\nu) = 50]$</td>
<td>45</td>
<td>0.95%</td>
<td>71.73%</td>
</tr>
<tr>
<td>Historical Simulation</td>
<td>55</td>
<td>1.16%</td>
<td>28.33%</td>
</tr>
</tbody>
</table>

Still, a correctly specified Value at Risk model should also be consistent in turbulent market environments. Therefore, we additionally account for volatility clustering and test our models on innovations of a GARCH(1,1) process\textsuperscript{21, 22}.

The results for the Kupiec test are displayed in table 5 and the percentage of hits per year in figure 5. Applying the five models on GARCH(1,1)-innovations rather than on the original return observations leads to a considerable improvement of the models’ performance. Only for the Variance-Covariance model the null hypothesis of a correct model specification can be rejected at the 5% significance level, while there is no rejection for the other models. Again, the historical simulation yields good results. These results reveal that applying Value at Risk models on GARCH-innovations rather than on empirical return observations and, hence, accounting for volatility clustering improves the quality of market risk measurements considerably. Although the non-parametric historical simulation approach performs as good as our three parametric models, it is important to note that the more one is going into the tails of return distributions, the more non-parametric models suffer from higher uncertainty in VaR estimates compared to parametric models (see, e.g. Aussenengg and Miazhynskaia [4]). This applies especially to the 99%-VaR and tends to generate an advantage for our new (parametric) copula-based approaches.

\textsuperscript{21}See Bollerslev [7] and Engle [13].

\textsuperscript{22}We are aware that the applied hit test is not strictly out-of-sample any more, as the GARCH(1,1) process is applied to the whole data sample at once.
Figure 5: Percentage of hits - GARCH(1,1)-innovations

Percentage of hits of the various models per year, applied to GARCH(1,1)-innovations. For a correctly specified model, the percentage should be 1%.

5 Conclusion

In this article we apply a hit test to evaluate the performance of a number of market risk models, using a data history of almost 20 years. For every trading day in our data history we estimate a one-day 99% Value at Risk (VaR) for an equally weighted portfolio consisting of 21 different financial assets. We use a rolling window of 250 trading days for calibration and compare these VaR estimates with the next day’s portfolio returns in a hit test (based on $n = 4,746$ daily observations).

The models employed are (a) the widely used Variance-Covariance model (as benchmark), (b) a meta-Gaussian copula model that allows for a more realistic modelling of asset returns, (c) the classical non-parametric historical simulation model, and (d) two variants of a meta-Student $t$ copula approach, for which we present a new parsimonious calibration procedure, allowing to model the probability of joint severe losses of financial assets.

The results of the hit test reveal that the null hypothesis of a correct model specification can be rejected for all models at the 5% significance level. The new meta-Student $t$ copula models and the historical simulation
model perform best, whereas the Variance-Covariance model performs worst. The overall rather poor performance of all models can be explained by severe risk underestimation during the financial crisis in 2008.

Applying the same models on GARCH(1,1)-innovations rather than on the original observations – and thus accounting for volatility clustering – leads to a considerable improvement of the models’ performance. The Variance-Covariance model is in this case the only model for which the hypothesis of a correct model specification has to be rejected at the 5% significance level.

We may hence conclude that the weaknesses of the widely used Variance-Covariance model stems from three sources: (a) an inappropriate modeling of the marginal distributions, i.e. the univariate asset return distributions, (b) an inappropriate modeling of the ‘dependence structure’, i.e. the copula, and (c) not accounting for volatility clustering. The newly presented meta-Student $t$ copula approach tends to overcome these weaknesses if volatility-clustering is accounted for.

The comparably good performance of the simple historical simulation model is noteworthy. However, also this model has weaknesses. As the estimation of the VaR is based on only 250 historical scenarios, the confidence level of the VaR cannot be higher than 99.6%. Hence, the applicability of the non-parametric historical simulation model tends to be less appropriate for lower return quantiles.

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